Inequality values and unequal shares

Anthony Shorrocks
UNU-WIDER, Helsinki
First draft, June 2005

1. Introduction

One of the great handicaps faced by researchers on inequality is the difficulty of conveying the significance of summary measures of inequality to a broad audience, especially non-economists. While concepts such as unemployment, inflation, growth, productivity and poverty can be grasped intuitively by the general public — although not with all the fine nuances — this is not the case with inequality values. The increasing attention given to issues concerning population heterogeneity has made the lack of an intuitive concept a more pressing problem. This perhaps explains a growing tendency to revert to the use of crude measures of inequality, such as the inter-decile ratio.

The Gini coefficient is the summary measure which comes closest to providing an intuitive interpretation. Indeed, this is the main reason why the Gini coefficient remains by far the most popular inequality index.[1] Yet the standard interpretations of the Gini coefficient fall far short of immediate comprehension. The most common interpretation is the area above the curve in a Lorenz diagram expressed as a proportion of the area below the diagonal; but this presupposes familiarity with the notion of a Lorenz curve. The Gini can also be defined in terms of the average absolute difference between incomes in the population, sampling randomly with replacement over the entire population. In fact Yitzhaki (1998) lists more than 12 alternative ways of defining the Gini coefficient — "spelling Gini" is how he puts it. However none of these linguistic variations succeed in providing the simple intuitive concept that everyone craves.

[1] It is not the only reason. The Gini is relatively insensitive to the tails of the distribution and hence relatively robust to problems associated with reliability of extreme values. The [0,1] range is another advantage, as is the network (or inertia) effect that favours indices used by others. The main complaint against the Gini coefficient is its failure to be "subgroup consistent".

The conception of inequality most easily understood is a division of a pie (or $1) into two unequal shares. Can the value of the Gini coefficient — or any other inequality index — be interpreted in this way? The answer is a qualified yes on all counts, and the relation with the Gini index turns out to be particularly simple. For example, the Gini value of 0.40 for inequality of US incomes in 2000 is precisely the same figure obtained from dividing $1 into 90c and 10c. The notion that current US income inequality is equivalent to a 2-way division of a pie in which one person gets 9 times the other is a powerful way of capturing the extent of income differences. For more equal countries, the 2 person income ratio is more modest: for Finland, for example, the Gini value of 0.27 translates into a split of $1 into 77c and 23c, a ratio of around 3:1.

These results follow from a remarkably simple property of the Gini coefficient that appears to have been overlooked before. Consider a division of 1 unit of resource between two people, in which the richer person gets x and the poorer person gets $1-x$. Since the "fair share" of each person is 50 per cent, we can regard $x-0.5$ as the "excess share" of the richer person. For the distribution $(x, 1-x)$ the Gini value turns out to be precisely $x-0.5$, in other words, the excess share of the rich person. Equivalently, the share of the rich person in a 2-way division can be expressed as $0.5+G$, where G is the Gini value. Hence the correspondence between the 90:10 ratio and the Gini value of 0.40 for the USA, and the 77:23 ratio and the Gini value of 0.27 for Finland.

A two-way split with non-negative shares yields a Gini coefficient between zero and one half.[2] It is not unusual for this upper bound to be exceeded, for instance for countries with high income inequality. For wealth inequality, Gini values above 50 per cent are commonplace. This limits the "excess share" interpretation of the Gini coefficient in a two-way split, unless negatives shares are entertained, which undermines the intuitive appeal of the interpretation. It turns out, however, that the excess share interpretation extends to any size of population with one rich person and the remainder equally poor. Here again the Gini coefficient equates to the excess share of the richer person. Thus for a 10 person distribution the fair share is 10 per cent and the Gini value G translates into a share of $0.1+G$ for the richest person and a share of a $0.1-G/9$ for each of the others. The Gini figure of 0.40 for US income inequality is therefore equivalent to

[2] This refers to the standard definition of the Gini coefficient, which is replication invariant. Researchers sometimes employ a variant that is normalised to the range [0,1] for populations of fixed finite size: See for example, Subramanian (2002).

a 10 person society in which one person receives 50 per cent of the pie and nine people each get 0.5/9 = 4.4 per cent.

The term “modulo10" is used to indicate excess shares expressed in terms of a 10-person distribution rather than a 2-person split (called “modulo2"). The advantage of shifting from modulo2 to modulo10 is that the feasible Gini range for non-negative incomes now extends to 0.9, which is high enough to accommodate almost all practical instances.

The Gini coefficient yields the neatest and most immediate interpretation in terms of the excess share of the rich in a 2-class society. However, the interpretation can be applied in principle to an appropriate normalisation of any inequality index. In effect one can ask: “according to this index, what 2-way split of the pie would generate the same inequality value as that observed”, and then compute the excess share of the richest person. The advantage of this procedure is that it converts all inequality values onto the same measuring scale, and hence allows the impact of changes in inequality perceptions to be seen more clearly. However, unlike the Gini, excess shares calculated modulo10 will typically differ from those calculated modulo2.

The comparison across inequality indices is illustrated in Section 3 with an application to country income distributions. Before then the basic framework of analysis is outlined in Section 2.

2. Calculating excess shares

For an n person population with incomes y_i arranged in increasing rank order so that $y_1 \le y_2 \le \ldots \le y_n$, the Gini coefficient may be expressed in the form:

$$\mathrm{G} = \frac{2}{n^2 \mu} \sum_{i=1}^{n} i\,(y_i - \mu) = \frac{2}{n} \sum_{i=1}^{n} i\,\frac{y_i}{n\mu} - \frac{n+1}{n}.$$

If the distribution consists of one rich person with the income share x and (n-1) poorer people each with income share (1-x)/(n-1), then the Gini value becomes

$$\mathrm{G} = \frac{2}{n} \sum_{i=1}^{n-1} i\,\frac{(1-x)}{(n-1)} + \frac{2}{n} n x - \frac{n+1}{n}$$

$$= \frac{2}{n}\ \frac{(n-1)n}{2}\ \frac{(1-x)}{(n-1)} + 2x - \frac{n+1}{n} = x - \frac{1}{n}$$

which corresponds exactly to the excess share of the richest person.

Similar expressions can be derived for other inequality indices. For example, the class of Entropy indices are given by

$$E_c = \frac{1}{n}\ \frac{1}{c(c-1)} \sum_{i=1}^{n} \left\{ \left(\frac{y_i}{\mu} \right)^c - 1 \right\}, \quad c \neq 0, 1$$

$$E_1 = \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\mu} \ln \frac{y_i}{\mu} \qquad \text{(Theil coefficient)}$$

$$E_0 = \frac{1}{n} \sum_{i=1}^{n} \ln \frac{\mu}{y_i} \qquad \text{(mean logarithmic deviation)}$$

while the Atkinson family of indices are given by

$$A_\alpha = 1 - \frac{1}{\mu} \left\{ \frac{1}{n} \sum_{i=1}^{n} y_i^{1-\alpha} \right\}^{1/(1-\alpha)} \qquad \alpha > 0,\ \alpha \neq 1,$$

$$A_1 = 1 - \frac{1}{\mu} (y_1 y_2 \ldots y_n)^{1/n}$$

If one again considers a distribution consisting of one rich person with the income share x and (n-1) poorer people each with income share (1-x)/(n-1), the values of each of these indices may be written as increasing functions of the excess share of the richest person, $x - 1/n$. However, the relationship the inequality value and the excess share is more complex and, as a consequence, the interpretation is less immediate.

3. Application to inequality comparisons between countries

To illustrate the conversion of inequality values into excess shares, data has been drawn from the latest version of the World Income Inequality Database (WIID). Country observations were selected by first choosing those for which decile shares are available for the distribution of consumption (preferred) or income (preferably net) across persons (rather than households) and which are representative of the whole population.[3] Restricting attention to the period from 1995 onwards reduced the sample to 78 countries from which the observation closest to the year 2000 was chosen.

To compute the inequality values, a utility constructed at WIDER was used to generate for each country a synthetic distribution of 1000 income values that exactly match the reported income decile data. Inequality values were then calculated for this synthetic sample and inverted to obtain the excess share figures modulo2 and modulo10.[4] For a subset of 30 countries, the raw decile data and Gini values are given in Table 1. Excess share values for a range of other indices are reported modulo2 and modulo10 in Tables 2 and 3, respectively.

From Table 2 it is immediately apparent that a number of the computed inequality values lie outside the feasible range for a two-person distribution, so the excess shares cannot be computed. This is particularly evident for the coefficient of variation (corresponding to E_2)[5]. This is a strong argument for focussing on the modulo10 results reported in Table 3, which all lie in the feasible range.

The second point to note in Table 2 (and also Table 3), is that the results for the entropy index E_{-1} are identical to those for the Atkinson index A_2, and the same correspondence is found for E_0 and A_1. There is a simple explanation. These pairs of indices are monotonic transformations of one another. The conversion into excess shares unravels this transformation, in effect applying the same measuring rod, and hence generating the same values. The fact that the excess share measuring rod produces identical results for ordinarily equivalent indices is one of the very attractive features of this procedure.

Comparing the results across indices in Table 2, it is evident that the values for the entropy and Atkinson indices are almost always less than the corresponding Gini figure. In this respect the Gini coefficient may be said to give an exaggerated impression of the degree of inequality in terms of the division of a pie between two people. This is a surprising outcome, and

---

[3] With minor omissions such as Chechnya in Russia.

[4] Elsewhere we have shown that for decile share data this method typically generates Gini estimates within 0.001 of the true value.

[5] Although note that this index can accommodate negative values, so the excess share of the richest person could in principle be extended beyond 50 per cent.

one for which there is no immediately obvious explanation.

The modulo10 values reported in Table 3 differ from the pattern of modulo2 values in Table 2. Excess shares modulo10 tend to be lower for relatively equal distributions, but higher for more unequal countries. The modulo10 figures are also higher everywhere for the indices that are most sensitive to the lower tail — the entropy index E_{-1} and its Atkinson counterpart A_2. Overall, the excess shares modulo10 again tend to be lower than the corresponding Gini value, although the difference is less marked, around 2 percentage points on average.

Alternative inequality indices apply different weights to different parts of the income distribution. One advantage of applying a common measuring rod to all indices is that it offers an opportunity to investigate how assessment of inequality varies as one moves, for example, from a concern with inequality at the bottom ("how do the poorest groups fare relative to the average") to a focus on the relative position of the rich. In particular it is interesting to look for cases in which the ranking of a pair of countries switches across indices, as these indicate instances when inequality assessments are likely to depend on the relative importance attached to the lower and upper tails.

Focussing on Table 3, for which a complete set of data is available, the modulo10 values for the Atkinson index increase monotonically as the degree of inequality aversion increases and hence more attention is given to the lower tail of the distribution. The entropy indices exhibit the same tendency as one moves from E_1 to E_{-1}. However, the pattern is not monotonic for all countries across the entire range of entropy indices. For some higher inequality countries, the excess share rises when E_1 is replaced by E_2, the latter being more sensitive to inequality in the upper part of the distribution.

For the set of countries as a whole, there are relatively few instances of significant re-ranking across the indices. This is because most pair-wise comparisons within the sample involve non-intersecting Lorenz curves so the inequality rankings are invariant to the choice of index. The most interesting exception is the comparison between South Africa and Brazil. The Gini coefficient is similar for the two countries. However, Brazil records an excess share value higher than the Gini for the index E_{-1}, suggesting that the Brazilian distribution is characterized by the relatively low incomes of the poor. In contrast, South Africa records a higher excess share for the coefficient of variation, suggesting a high degree of inequality at the very top. This is exactly the position indicated by the decile share figures in Table 1.

References

Duclos, J-Y. and A. Araary (2005): "Poverty and Equity: Measurement, Policy and Estimation with DAD", Preliminary version, downloadable pdf file.

Subramanian, S. (2002): "An elementary interpretation of the Gini inequality index", *Theory and Decision*, 375-379.

Yitzhaki, S. (1998): "More than a dozen ways of selling Gini", *Research on Economic Inequality*, **8**, 13-30.

Table 1: Decile Shares, selected countries

Country	Year	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	Gini
Czech Republic	1996	4.0	5.8	6.8	7.7	8.5	9.2	10.2	11.6	13.9	22.4	26.1
Finland	2000	4.0	5.6	6.6	7.5	8.3	9.2	10.3	11.8	14.1	22.6	26.8
Sri Lanka	2000	3.7	5.4	6.5	7.4	8.3	9.2	10.4	12.0	14.3	22.7	27.6
Germany	2000	3.4	5.2	6.3	7.2	8.2	9.3	10.5	12.3	14.9	22.8	29.0
Hungary	1999	3.3	5.0	6.1	7.1	8.2	9.2	10.3	11.8	14.3	24.8	30.5
Netherlands	1999	2.5	5.1	6.1	7.1	8.1	9.2	10.6	12.6	15.8	22.8	30.7
Canada	2000	2.7	4.6	5.8	6.9	8.0	9.1	10.6	12.4	15.1	24.8	32.4
Italy	2000	2.1	4.1	5.4	6.5	7.7	9.0	10.5	12.3	15.3	26.9	35.8
India	2000	3.2	4.4	5.3	6.1	7.1	8.2	9.7	11.8	15.4	28.9	36.0
Indonesia	1996	3.3	4.4	5.3	6.1	7.0	8.1	9.4	11.4	14.8	30.1	36.5
United Kingdom	1999	2.5	4.1	5.1	6.2	7.3	8.6	10.1	12.2	15.5	28.4	37.0
Bangladesh	1996	2.9	4.2	5.1	6.0	6.9	7.9	9.3	11.5	15.0	31.2	38.2
United States	2000	1.8	3.5	4.7	5.9	7.2	8.5	10.1	12.3	16.0	29.9	40.1
Ukraine	1995	2.2	3.6	4.7	5.7	6.8	8.0	9.5	11.8	15.2	32.3	41.1
Philippines	1997	2.5	3.5	4.4	5.3	6.4	7.7	9.4	11.9	16.0	32.9	42.2
Cambodia	1999	2.4	3.6	4.6	5.3	6.0	7.0	8.3	10.5	15.0	37.1	44.5
Thailand	1999	2.1	3.2	4.1	5.0	6.1	7.4	9.3	12.1	16.7	34.0	44.6
China	1995	1.9	3.1	3.9	4.9	6.0	7.6	9.6	12.5	16.9	33.7	45.2
Russia	2000	1.0	2.8	4.2	5.6	6.8	8.0	10.0	11.6	15.6	34.4	45.3
Venezuela	2000	1.4	2.8	3.9	5.1	6.3	7.8	9.7	12.4	16.8	33.8	45.8
Uganda	1999	2.1	3.2	4.1	4.9	5.9	7.0	8.5	10.8	15.3	38.3	46.9
Nigeria	1996	1.7	2.9	3.8	4.7	5.7	7.0	8.8	11.4	16.0	38.0	48.3
Malaysia	1995	1.6	2.6	3.5	4.5	5.5	6.9	8.7	11.4	16.2	39.1	50.0
Ghana	1998	0.8	2.1	3.2	4.4	5.9	7.5	9.6	12.3	16.9	37.2	50.6
Mexico	2000	1.1	2.1	3.1	4.1	5.2	6.7	8.4	11.0	16.4	41.9	53.5
El Salvador	2000	0.7	1.8	2.9	4.1	5.4	6.9	8.9	11.8	17.3	40.1	53.8
Egypt	1997	1.2	2.4	3.3	4.2	5.2	6.3	7.8	10.1	14.6	44.8	54.2
Chile	2000	0.9	2.0	2.7	3.5	4.4	5.5	7.0	9.5	14.6	49.7	59.4
South Africa	1997	1.3	2.3	3.0	3.7	4.4	5.1	6.2	8.0	11.8	54.3	60.2
Brazil	2001	0.8	1.6	2.4	3.2	4.2	5.2	7.0	9.8	15.7	50.0	61.1
Zimbabwe	1995	0.5	1.1	1.5	2.1	2.8	3.6	4.7	6.4	9.9	67.4	73.3

Table 2: Excess Shares Modulo 2

Country	Gini	Entropy Index				Atkinson Index			
		-1	0	1	2	0.5	1.0	1.5	2.0
Czech Republic	26.1	22.3	22.5	24.0	27.3	23.1	22.5	22.3	22.3
Finland	26.8	22.4	22.9	24.4	27.6	23.5	22.9	22.5	22.4
Sri Lanka	27.6	23.4	23.6	25.0	28.2	24.2	23.6	23.4	23.4
Germany	29.0	25.3	24.9	25.9	28.8	25.2	24.9	24.9	25.3
Hungary	30.5	25.7	25.9	27.9	32.6	26.6	25.9	25.6	25.7
Netherlands	30.7	29.1	26.3	27.5	29.2	25.3	26.3	28.2	29.1
Canada	32.4	28.3	27.6	29.0	33.0	28.0	27.6	27.7	28.3
Italy	35.8	36.4	30.6	32.2	37.7	30.5	30.6	32.1	36.4
India	36.0	28.0	29.2	32.7	40.4	30.6	29.2	28.4	28.0
United Kingdom	37.0	30.3	30.5	33.2	40.4	31.4	30.5	30.1	30.3
Bangladesh	38.2	29.4	30.8	35.2	46.1	32.5	30.8	29.8	29.4
United States	40.1	33.3	33.2	35.8	43.8	33.8	33.2	33.5	33.3
Ukraine	41.1	32.4	33.1	37.4	49.6	34.6	33.1	32.4	32.4
Philippines	42.2	32.0	33.3	37.9	49.7	35.1	33.3	32.4	32.0
Cambodia	44.5	32.6	34.7	41.1	***	37.1	34.7	33.3	32.6
Thailand	44.6	33.8	34.9	39.5	***	36.6	34.9	34.1	33.8
China	45.2	39.7	35.8	39.8	***	37.1	35.8	36.0	39.7
Russia	45.3	31.3	36.8	40.6	***	37.1	36.8	38.0	31.3
Venezuela	45.8	29.6	36.5	40.3	***	37.6	36.5	36.5	29.6
Uganda	46.9	34.3	36.1	42.7	***	38.5	36.1	34.8	34.3
Nigeria	48.3	36.2	37.2	43.1	***	39.3	37.2	36.3	36.2
Malaysia	50.0	36.9	38.1	44.3	***	40.3	38.1	37.1	36.9
Ghana	50.6	40.3	39.7	44.2	***	40.7	39.7	40.4	40.3
Mexico	53.5	39.9	40.3	46.8	***	42.5	40.3	39.6	39.9
El Salvador	53.8	44.0	41.4	46.5	***	42.6	41.4	42.1	44.0
Egypt	54.2	39.5	40.4	48.4	***	43.1	40.4	39.3	39.5
Chile	59.4	42.6	42.7	***	***	45.7	42.7	41.8	42.6
South Africa	60.2	39.6	42.5	***	***	46.4	42.5	40.4	39.6
Brazil	61.1	42.1	43.4	***	***	46.2	43.4	42.2	42.1
Zimbabwe	73.3	45.0	47.0	***	***	***	47.0	45.4	45.0

Table 3: Excess Shares Modulo 10

Country	Gini	Entropy Index				Atkinson Index			
		-1	0	1	2	0.5	1.0	1.5	2.0
Czech Republic	26.1	23.5	19.7	17.4	16.4	18.4	19.7	21.4	23.5
Finland	26.8	23.8	20.1	17.7	16.5	18.8	20.1	21.8	23.8
Sri Lanka	27.6	25.4	21.0	18.3	16.9	19.5	21.0	23.0	25.4
Germany	29.0	28.5	22.6	19.1	17.3	20.6	22.6	25.2	28.5
Hungary	30.5	29.3	24.0	20.8	19.6	22.1	24.0	26.4	29.3
Netherlands	30.7	35.7	24.5	20.4	17.5	20.7	24.5	30.5	35.7
Canada	32.4	34.1	26.3	21.8	19.8	23.7	26.3	29.8	34.1
Italy	35.8	51.6	30.6	24.9	22.6	26.6	30.6	37.6	51.6
India	36.0	33.5	28.6	25.4	24.3	26.7	28.6	30.9	33.5
United Kingdom	37.0	38.1	30.4	25.9	24.3	27.7	30.4	33.9	38.1
Bangladesh	38.2	36.2	31.0	27.9	27.6	29.1	31.0	33.4	36.2
United States	40.1	44.5	34.8	28.4	26.3	30.8	34.8	40.5	44.5
Ukraine	41.1	42.5	34.7	30.2	29.8	31.9	34.7	38.3	42.5
Philippines	42.2	41.7	35.1	30.7	29.8	32.5	35.1	38.3	41.7
Cambodia	44.5	42.9	37.4	34.4	35.6	35.4	37.4	40.0	42.9
Thailand	44.6	45.6	37.8	32.5	31.1	34.7	37.8	41.6	45.6
China	45.2	59.8	39.5	32.8	31.1	35.4	39.5	45.8	59.8
Russia	45.3	40.2	41.3	33.8	31.9	35.4	41.3	50.3	40.2
Venezuela	45.8	36.6	40.7	33.4	31.4	36.2	40.7	46.9	36.6
Uganda	46.9	46.7	40.0	36.3	37.5	37.6	40.0	43.2	46.7
Nigeria	48.3	51.2	42.1	36.8	37.2	38.8	42.1	46.4	51.2
Malaysia	50.0	53.0	43.9	38.3	38.9	40.4	43.9	48.3	53.0
Ghana	50.6	61.5	47.4	38.2	36.7	41.2	47.4	56.4	61.5
Mexico	53.5	60.4	48.7	41.8	43.0	44.3	48.7	54.4	60.4
El Salvador	53.8	71.8	51.2	41.4	40.6	44.5	51.2	61.1	71.8
Egypt	54.2	59.5	48.9	44.4	49.1	45.6	48.9	53.7	59.5
Chile	59.4	67.9	54.7	50.1	57.4	51.1	54.7	60.4	67.9
South Africa	60.2	59.6	54.0	54.2	67.5	52.9	54.0	56.6	59.6
Brazil	61.1	66.3	56.4	50.9	57.0	52.5	56.4	61.5	66.3
Zimbabwe	73.3	74.6	68.3	70.5	89.9	67.2	68.3	71.4	74.6