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Poverty Measures and Anti-Poverty Policy with an Egalitarian Constraint

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Abstract

Bourguignon and Fields ('Poverty Measures and Anti-Poverty Policy') and Gangopadhyay and Subramanian ('Optimal Budgetary Intervention in Poverty Alleviation Schemes') have derived optimal budgetary rules for the redress of poverty through direct income transfers when poverty is measured by the Foster, Greer and Thorbecke P_α class of indices in the context of a constrained optimization exercise which one may call the 'canonical problem'. For the stock of poverty measures most commonly in use, the canonical problem yields one of only three types of optimal solution which Bourguignon and Fields call, respectively, a 'type-r', a 'type-p' and a 'mixed-type' policy. The authors suggest that other types of policy—such as a 'proportionality rule' in which the budget is allocated in proportion to a poor person's share in the aggregate poverty deficit—could be considered optimal in certain circumstances. This paper explores the sorts of circumstances—meaning combinations of poverty indices and different types of 'egalitarian' restrictions on optimal transfers—under which 'unconventional' policies (including the Proportionality Rule) could emerge as optimal budgetary allocations.

Keywords: poverty, transfer schedules, income-gap ratios

JEL classification: I32, I38, D63

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1 Introduction

When poverty is sought to be alleviated through direct income transfers, and assuming perfect knowledge regarding the personal distribution of incomes, the question arises as to how best to allocate a budget of given size among competing contenders for it from among the poor population. This problem has been considered by, among others, Bourguignon and Fields (1990)—hereafter BF—and Gangopadhyay and Subramanian (1992)—hereafter GS. The optimal solution, as might be expected, would depend on the precise way in which one measures poverty, and also on the constraints that constitute part of the optimization exercise. Both BF and GS consider the Foster, Greer and Thorbecke (1984)—hereafter FGT—P_α family of poverty indices, while BF, additionally, consider Sen's (1976) poverty index as well. The constrained optimization problem addressed by both sets of authors is what one might call the *canonical problem*. In the canonical problem, poverty is sought to be minimized, by appropriate choice of an income-transfer schedule, subject to a budget constraint, and a floor (non-negativity) and a ceiling (poverty-gap) constraint on the transfers. The principal objective of this paper is to analyse a slight complication of the canonical problem by allowing for a tighter constraint structure through the specification of certain egalitarian restrictions on the income-transfers.

For the canonical problem considered by BF and GS, and given the stock of poverty indices most commonly in use, the optimal transfer schedule assumes one or other of three forms which BF call, respectively, a 'type-r' policy, a 'type-p' policy and a 'mixed-type' policy. A 'type-r' policy is a 'type of anti-poverty policy that transfers all of the available budget to the richest of the poor…' (BF 1990: 413). A 'type-p' policy is one which 'does the opposite. Only the poorest of the poor receive a transfer, which brings them all up to the same income level, still below the poverty line…' (BF 1990: 414). A 'mixed-type' policy is one which requires the allocation of 'a (strictly positive) fraction of the money to the poorest of the poor' (BF 1990: 416).

The BF paper ends on a note of presenting an interesting problem for further research. The authors point out that, under certain circumstances, one may wish the optimal anti-poverty policy to be different from every one of the type-r, type-p and mixed-type allocations. A plausible optimal solution, for example, is an allocation in which each poor person receives a share of the budget in proportion to the shortfall of the person's income from the poverty line. These sorts of optimal solutions would presumably be called forth by optimization problems that go beyond the canonical problem and which entail the additional postulation of certain 'equality-preferring' constraints. This paper considers a few such 'expanded' problems and the corresponding solutions for the FGT class of poverty indices.

The paper is organized as follows. Section 2 presents preliminary formalities of concepts and definitions. Section 3 briefly reviews the canonical problem and its solution for the FGT family of poverty measures. Section 4 discusses transfer schedules different from 'type-r', 'type-p' and 'mixed-type' schedules which might have a measure of *prima facie* plausibility attached to them. Sections 5 and 6 analyse the solutions for the FGT poverty indices in the context of two 'expanded' versions of the canonical problem, incorporating certain additional 'egalitarian' constraints. Section 7 presents a simple numerical example to elucidate the results of the preceding sections. Concluding observations are offered in section 8.

2 Basic concepts

For every positive integer n, let $\mathbf{X}^n$ be the set $\{\mathbf{x}\in \boldsymbol{S}^n \mid x_i \leq x_{i+1}, i = 1,\ldots,n-1\}$, where $\boldsymbol{S}^n$ is the non-negative orthant of n-dimensional real space. Every $\mathbf{x}$ belonging to $\mathbf{X} \equiv \cup_{n=1}^{\infty}\mathbf{X}^n$ is then a description of an n-person ordered distribution of (non-negative) incomes. The *poverty line*, which is a positive level of income such that any person whose income is below this level is certified to be *poor*, is designated by z. $\boldsymbol{T}$ will stand for the set of positive, and $\boldsymbol{S}$ for the set of non-negative, reals. The set of poor persons is $Q \equiv \{i \mid x_i < z\}$. The cardinality of Q will be denoted by q. A *poverty index* is a mapping $P:\mathbf{X}\text{x}\boldsymbol{T} \rightarrow \boldsymbol{S}$ such that, for every regime ($\mathbf{x}$;z) of income-distribution and positive poverty line, P assigns a real number which is intended to signify the extent of poverty in the regime ($\mathbf{x}$;z). Two commonly invoked desirable properties of poverty indices are the following (see Sen 1976 and Foster 1984):

Monotonicity (Axiom M). Other things equal, an increase in a poor person's income should reduce poverty.

Transfer (Axiom T). Other things equal, a rank-preserving transfer of income from a poor person to a poorer person should reduce poverty.

For every ($\mathbf{x}$;z) in the domain of the poverty function, the *Foster-Greer-Thorbecke, or P_α, class of poverty measures*, is given by:

(2.1) $P_\alpha(\mathbf{x};z) = (1/n)\Sigma_{i=1}^{q}[(z-x_i)/z]^\alpha, \alpha \geq 0.$

When $\alpha = 0$, we obtain the *headcount ratio* $H \equiv q/n$. When $\alpha = 1$, we obtain the *per capita income-gap ratio* $R \equiv HI$, where $I \equiv (z-\mu^P)/z$ is the income-gap ratio and μ^P is the average income of the poor. When $\alpha = 2$, we obtain the 'distributionally sensitive' poverty index given by $P_2 = H[I^2 + (1 - I)^2C^2]$ where C^2 is the squared coefficient of variation in the distribution of poor incomes. As α becomes larger and larger, P_α becomes more and more distributionally sensitive until, in the limit, as $\alpha \rightarrow \infty$, P_α converges on a sort of Rawlsian 'maximin' measure, in terms of which the poverty ranking of distributions is determined solely by the income of the poorest person.

It may be noted that the P_α family of indices belongs to the class of 'additively separable poverty measures' (see Atkinson 1987 and Keen 1992): each index P in this class can be written as a simple average of the 'deprivation functions' $\phi(x_i;z)$ of all individuals, where ϕ displays the property that $\phi(x_i;z) > 0$ for $x_i < z$ and $\phi(x_i;z) = 0$ for $x_i \geq z$. For the P_α family of indices, the relevant deprivation functions ϕ_α are given by

$$\phi_\alpha(x_i;z) = [(z - x_i)/z]^\alpha \ \forall i\in Q, \alpha \geq 0$$
$$= 0 \ \forall i\notin Q$$

If we plot ϕ_α against income, it can be checked that ϕ_α is a constant function of income over the range [0,z) when $\alpha = 0$; ϕ_α is a declining, *strictly concave* function for $\alpha\in (0,1)$; ϕ_α is a declining *linear* function for $\alpha = 1$; and ϕ_α is a declining, *strictly convex* function for $\alpha > 1$. As is well known, the poverty index P satisfies Axiom M whenever ϕ is strictly declining, and P satisfies Axiom T whenever ϕ is declining *and* strictly convex. Thus, P_0 violates both Monotonicity and Transfer; for $\alpha\in (0,1)$, P_α satisfies

Monotonicity but violates Transfer—in fact it *rewards* regressive transfers among the poor; P_1 satisfies Monotonicity but fails Transfer (it is invariant with respect to interpersonal redistributuions of income among the poor); and for $\alpha > 1$, P_α satisfies both Axioms M and T. These features of the members of the P_α family of measures are reflected in the corresponding optimal anti-poverty transfer schedules associated with them, as will be seen in the following section.

3 The canonical problem and its solution for the P_α indices

The 'canonical' or 'standard' way in which the constrained poverty-minimization problem is formulated, and which will be called 'problem 1', is set out below, in the specific context of the P_α class of poverty indices. In what follows, T is the budget available for transfers to the poor, and it is assumed that $z \leq T < D$, where $D \equiv \Sigma_{i=1}^{q}(z - x_i)$ is the *aggregate poverty deficit*, namely the total income that would be required to extricate the entire poor population from poverty. The lower bound on T indicates that the budget is always large enough to lift at least one 'completely' (i.e., zero-income) poor person out of poverty: this is purely an assumption of convenience, designed to avoid clutter in the subsequent presentation of optimal solutions. The upper bound on T—the requirement that $T < D$—is to keep the problem of poverty-redress non-trivial. It will also throughout be assumed that an incremental rupee allocated to the poor which does not reduce poverty in terms of the chosen poverty measure will not be spent on the poor: that incremental rupee is more efficiently spent on other competing claims such as infrastructure, or import-substitution, or whatever.

Problem 1

Minimize $P_\alpha(x_1 + t_1,\ldots,x_q + t_q) = (1/n)\Sigma_{i=1}^{q}[(z - x_i - t_i)/z]^\alpha$, $\alpha \geq 0$
$\{t_1,\ldots,t_q\}$

subject to

(A) $\Sigma_{i=1}^{q} t_i \leq T$

(B) $t_i \leq z - x_i \ \forall i = 1,\ldots,q$

(C) $t_i \geq 0 \ \forall i = 1,\ldots,q$

Constraint A is a simple feasibility constraint which says that the sum of transfers to the poor should not exceed the budgetary outlay available. Constraint B imposes a measure of 'efficiency' on the individual transfers, by requiring that no resources should be wasted on any individual by allocating to her a transfer in excess of her poverty gap. Constraint C is a non-negativity constraint which says that no poor person may be taxed.

The solutions to problem 1 for the P_α class of indices, in each of four cases ($\alpha = 0$, $\alpha \in (0,1)$, $\alpha = 1$, and $\alpha > 1$), are provided below. These are essentially the results presented in BF (1990) and GS (1992), and will therefore not be proved here. In what follows, s will stand for the smallest integer such that $\Sigma_{i=s}^{q}(z - x_i) \leq T$, and b will stand for the largest integer such that $x_b \leq x^*$, where x^* is that level of income which satisfies $\Sigma_{i=1}^{b}(x^* - x_i) = T$. For parametric variation of α in the P_α family of poverty measures, the corresponding optimal transfer schedules $\{t_i^*\}_{i \in Q}$ look like this:

$\alpha = 0$

$t_i^* = 0\ \forall i = 1,\ldots,s - 1$

$\quad = z - x_i\ \forall i = s,\ldots,q$

$\alpha \in (0,1)$

$t_i^* = 0\ \forall i = 1,\ldots,s - 2$

$\quad = T - \Sigma_{i=s}^{q}(z - x_i)$ for $i = s - 1$

$\quad = z - x_i\ \forall i = s,\ldots,q$

$\alpha = 1$

If **B** is the set of all allocations which exhaust the budget while satisfying the non-negativity constraint, then every $\{t_i\}_{i\in Q}\in \mathbf{B}$ is optimal. That is to say, any feasible transfer schedule which exhausts the budget is also optimal.

$\alpha > 1$

$t_i^* = x^*\text{-}x_i\ \forall i = 1,\ldots,b$

$\quad = 0\ \forall i = b + 1,\ldots,q$

As can be seen from the above, the solutions for $\alpha = 0$ and $\alpha\in (0,1)$ are what BF refer to as a 'type-r' policy, which is essentially a 'top-down' stratagem of 'skimming off the surface'. It may be noted that s could be such that $\Sigma_{i=s}^{q}(z - x_i) < T$: since P_0 does not satisfy Axiom M, there would be no point in allocating any part of the unutilized budget to the remaining $(q - s)$ poor persons. However, for $\alpha\in (0,1)$, P_α does satisfy Monotonicity; arising from this, the optimal transfer schedule must exhaust the budget (see GS, 1992), which is why the quantity $[T - \Sigma_{i=s}^{q}(z - x_i)]$ is allocated to individual $s - 1$. By way of stressing a minor difference, we may wish to call the solution for $\alpha = 0$ a 'type-r' policy, and the solution for $\alpha\in (0,1)$ a 'type-r+' policy. (Of course, if s were such that $\Sigma_{i=s}^{q}(z - x_i) = T$, then the solutions for $\alpha = 0$ and $\alpha\in (0,1)$ would be identical: r would coincide with r+). When $\alpha = 1$, one has an infinite number of solutions of which 'type-r', 'type-p' and 'mixed-type' policies are all special cases: this is not a particularly helpful guide to policy. Finally, for $\alpha > 0$, the optimal policy is a 'type-p' one, in terms of which the beneficiaries are the poorest of the poor, all of whom are brought up to a common level of income (x^*) below the poverty line. This solution implements a sort of 'lexicographic maximin' principle.

The next section briefly considers possible anti-poverty policies which are different from 'type-r', 'type-p' and 'mixed-type' policies.

4 Alternative plausible anti-poverty policies

BF (1990) conclude their paper on the following note:

> Following the argument in this paper, an alternative approach to the axiomatics of poverty would be to start from assumptions about the optimal allocation of an anti-poverty budget. Usual measures all lead to optimal allocations of type-p, type-r or mixed. But one may consider that another type of allocation should be optimal in some circumstances. For instance, the budget T could be optimally transferred to every one proportionally to his/her poverty shortfall [$(z - x_i)$]. The problem would then be to characterize all poverty measures consistent with such optimal allocations and some other basic axioms.

Let us call a 'proportionality rule' of the type described by BF as a 'PR-type' policy. Are there other rules, outside the 'type-p', 'type-r' and 'mixed-type' rules, which have at least a measure of plausibility, at first glance, attached to them? Here are a couple of such rules. One type of policy may be based on the notion that the only meaningful redress of the condition of poverty is to actually lift a person out of poverty, and further, that the greatest urgency attaches to raising the poorest of the poor to the poverty line: such a policy will be called a 'type-b' policy, to signify that it entails a 'bottom-up' approach to poverty alleviation. A second type of policy may be premised on the view that all poor persons have an equal claim on any incremental budgetary resource that may be available for distribution among them: such an 'equal division' policy could be called a 'type-e' policy.

Notice that a type-e policy is 'egalitarian' only to the extent that it prohibits the poorer of two poor individuals from receiving a smaller share of the budget. Indeed, if **E** is the class of all such 'weakly egalitarian' solutions—namely solutions which demand that the poorer of two poor persons should never receive a smaller share of the budget—then it is clear that type-b, type-p, PR-type and type-e policies all belong to this class. Indeed, these solutions occupy decreasingly 'pro-poorest-of-the-poor' positions in a spectrum of 'egalitarian' policies which is bounded on the 'right' by the type-e policy. As we proceed from a type-b policy to a type-e policy, we move in a direction which is less and less influenced by a concern for rectifying the existing structure of inequality in determining optimal budgetary intervention.

A type-b policy so strongly favours the poorest of the poor that it ends up, arguably, overcompensating in their behalf, by requiring the budgetary allocation to be such that in the post-transfer regime the poorest of the poor actually overtake their initially more fortunate cohorts. This has the appearance of amounting less to a rectification of inequality than to a retribution for it. Type-p, which we have seen is based on a lexicographic maximin principle, is less extreme; even so, 'Rawls-type' rules have sometimes been criticized for insisting on a stringent form of egalitarianism, whereby 'the dictatorship of the weakest/poorest' is upheld. The PR-type policy clearly adopts a more (respectively, less) relaxed stance in asserting the claims of the poorer among the poor than type-p (respectively, type-e). Type-e itself, as we have seen, defers to the interests of the most disadvantaged only to the extent that it disallows the less poor of two poor persons from obtaining a larger share of the budget. Of course, once solutions of the type-r variety are considered, it is transparent that the boundary of the class **E** of solutions has been transgressed, and one is dealing with solutions which are plainly inegalitarian.

When the canonical problem is addressed, the only anti-poverty policy belonging to the class **E** of 'weakly egalitarian' policies which is known to have emerged as a solution, is the type-p rule. To obtain other egalitarian solutions of the types ('b', 'PR', and 'e') considered in the immediately preceding paragraphs, the canonical problem must presumably be expanded by placing some additional 'equality-preferring' restrictions on the optimal transfer schedule. After all, the policy maker's 'values' are reflected not only in the objective function (in this case the poverty measure) of the optimization problem, but also in the way in which s/he chooses to specify the constraints under which the extremization exercise is carried out. In the canonical problem—see the specification of problem 1 in section 3—it is true that Constraint A is not much more than a requirement of sanity; but Constraints B and C are not in this sense value-neutral. Constraint B expresses a preference for an 'efficient' or 'unwasteful' use of resources, while Constraint C expresses a value regarding the political/ethical unsustainability of taxing the poor. It would thus appear to be unexceptionable to add to the constraints of the canonical problem, to make these reflect, for example, a preference for allocational outcomes which lean in the direction of favouring the poorer among the poor. Of course, any such constraint should be stated in such a way that, in a non-trivial sense, it falls short of actually explicitly stipulating a unique outcome. Two such constraints are presented below.

First, some notation. For every $i \in Q$, let $d_i \equiv (z - x_i)$ be person i's shortfall from the poverty line. For all $j,k \in Q$, define the index of pairwise inequality, $\sigma_{j,k}$, as the share of j's income-gap in the combined income-gaps of both j and k, namely $\sigma_{j,k} \equiv d_j/(d_j + d_k)$. Clearly, $\sigma_{j,k} \geq ½$ if $d_j \geq d_k$, and $\sigma_{j,k} < ½$ if $d_j < d_k$. Finally, for all $j,k \in Q$, let $\sigma'_{j,k}$ be the value of $\sigma_{j,k}$ *after* the transfer has been effected. Two 'egalitarian' constraints, in addition to the constraints embodied in the canonical problem, which could be considered are Constraints D and D′, as specified below.

Constraint D. For all $j,k \in Q$, if $x_j \leq x_k$, then $t_j \geq t_k$.

Constraint D simply states that the poorer of two poor persons should not get a smaller share of the budget.

Constraint D′. For all $j,k \in Q$, if $\sigma_{j,k} \geq ½$, then $\sigma'_{j,k} \leq \sigma_{j,k}$.

According to Constraint D′, the optimal transfer schedule should respect the requirement that for the poorer of two poor persons, the index of pairwise inequality $\sigma_{j,k}$ should not worsen.

By adding Constraints D and D′ respectively to the canonical problem (or problem 1), we obtain a couple of expanded versions of the standard optimization exercise, which can be called 'problem 2' and 'problem 3', respectively:

Problem 2

$$\underset{\{t_1,\ldots,t_q\}}{\text{Minimize}}\ P_\alpha(x_1 + t_1,\ldots,x_q + t_q) = (1/n)\Sigma_{i=1}^{q}[(z - x_i - t_i)/z]^\alpha,\ \alpha \geq 0$$

subject to

(A) $\Sigma_{i=1}^{q} t_i \leq T$

(B) $t_i \leq z - x_i \ \forall i = 1,\ldots,q$

(C) $t_i \geq 0\ \forall i = 1,\ldots,q$

(D) For all $j,k \in Q$, if $x_j \leq x_k$, then $t_j \geq t_k$

Problem 3

Minimize $P_\alpha(x_1 + t_1,\ldots,x_q + t_q) = (1/n)\Sigma_{i=1}^{q}[(z - x_i - t_i)/z]^\alpha$, $\alpha \geq 0$
$\{t_1,\ldots,t_q\}$

subject to

(A) $\Sigma_{i=1}^{q} t_i \leq T$

(B) $t_i \leq z - x_i\ \forall i = 1,\ldots,q$

(C) $t_i \geq 0\ \forall i = 1,\ldots,q$

(D′) For all $j,k \in Q$, if $\sigma_{j,k} \geq ½$, then $\sigma'_{j,k} \leq \sigma_{j,k}$

The optimal solutions to Problems 2 and 3, when the P_α class of poverty indices is employed, are considered in Sections 5 and 6 respectively.

5 Optimal solutions to problem 2 for the P_α class of indices

$\alpha = 0$

When $\alpha = 0$, the object is to minimize the headcount ratio. Clearly, the appropriate stratagem cannot be one of bridging the income-gaps of the richest of the poor: since the index H fails the monotonicity axiom, and since the budget T is smaller than what it takes—namely the quantity D—to wipe out poverty, the poorer of the poor would end up receiving no transfer—which falls foul of Constraint D. Minimization of the headcount ratio would now therefore require that we start by bridging the income-gap of the poorest person and work our way upward until the budget is exhausted. Formally, the optimal allocation is of the following type. If c is the largest integer such that $\Sigma_{i=1}^{c}(z - x_i) \leq T$, then

$t_i^* = z - x_i\ \forall i = 1,\ldots,c$

$\quad = 0\ \forall i = c + 1,\ldots,q$

This, precisely, is what has earlier been alluded to as a ‘type-b’ policy.

$\alpha \in (0,1)$

As has been noted earlier, when $\alpha \in (0,1)$, the deprivation function ϕ_α is a declining, *strictly concave* function of income: consequently, poverty minimization would favour larger allocations to the *richer* among the poor. Starting from the richest of the poor individuals—individual q—it must first be determined as to what is the largest transfer that can be made to q. Given Constraint D, any allocation made to q cannot be larger than the allocation made to $q - 1$, which in turn cannot be larger than the allocation made to $q - 2$, and so on down the line. Under these circumstances, it is clear that the largest allocation that can be made to q is T/q, *provided* T/q does not exceed $(z - x_q)$: this proviso is necessitated by respect for Constraint B. If T/q does not exceed $(z - x_q)$, then obviously the optimal allocation is one in which each poor person receives a transfer of T/q. If, however, T/q exceeds $(z - x_q)$, the largest amount q can receive is $(z - x_q)$. Turning next to individual $q - 1$, we compare the quantities $[T - (z - x_q)]/(q - 1)$ and $(z - x_{q-1})$: if the former does not exceed the latter, it would be optimal to allocate a transfer of $[T - (z - x_q)]/(q - 1)$ to each of the poorest $q - 1$ individuals; if the former does exceed the latter, person $q - 1$ should be allocated an amount of $(z - x_{q-1})$. We proceed with this sequence of calculations down the line from richest (of the poor) to poorest individual in order to arrive at the overall optimal solution, which can be stated in terms of the following recursive structure of transfers:

$$t_i^* = \min[(z - x_i), \{(T - \Sigma_{j=i+1}^{q} t_j^*)/i\}]\ \forall i = 1,\ldots,q - 1$$

and

$$t_q^* = \min[(z - x_q), T/q].$$

This, precisely, is what has earlier been alluded to as a 'type-e' policy.

$\alpha = 1$

As in the canonical problem, there is no unique solution for minimizing P_1 in problem 2: the set of optimal solutions for problem 2 is a strict subset of the set of optimal solutions for problem 1, and consists of all feasible allocations which exhaust the budget *and* respect the requirement that a poorer individual never receives a smaller share of the budget than a richer poor individual. Thus, each of the type-b, type-p, PR-type and type-e policies—indeed, every budget-exhausting allocation in the class **E** of policies—is an optimal P_1–minimizing rule for problem 2. However, a type-r+ policy which is optimal for problem 1 is no longer so for problem 2.

$\alpha > 1$

It is obvious from inspection that the type-p policy is also the optimal solution for problem 2 when $\alpha > 1$: Constraint D has no 'effective bite' that might necessitate a deviation from a type-p policy.

6 Optimal solutions to problem 3 for the P_α class of indices

$\alpha = 0$

Mere inspection assures us that the type-b solution is again the optimal solution for problem 3.

$\alpha \in (0,1)$

Recalling the definitions of $\sigma_{j,k}$ and $\sigma'_{j,k}$, it is easy to verify that Constraint D′ can be written equivalently as: For all $j,k \in Q$, if $x_j \le x_k$, then $t_j/(z - x_j) \ge t_k/(z - x_k)$. By virtue of strict concavity of the deprivation function ϕ_α for all values of α in the interval (0,1), the value of the poverty index declines more, for any given income transfer, the richer is the beneficiary. Consistent with this would be to set $t_j/(z - x_j) = t_k/(z - x_k)$ for all j,k such that $x_j \leq x_k$, that is, to set $t_i/(z - x_i) = \lambda$ (say) for all $i \in Q$, where λ is any real number. Since $P_{\alpha\in(0,1)}$ satisfies Monotonicity, the optimal solution $\{t_i^*\}$must be one which exhausts the budget. That is, since $t_i^*/(z - x_i) = \lambda$ for all $i \in Q$, we require that $(\Sigma_1^q\ t_i^* =)\ \lambda\Sigma_1^q(z - x_i) = T$, or $\lambda = T/D$, where $D \equiv \Sigma_1^q(z - x_i)$ is, as noted earlier, the aggregate poverty deficit. Stated formally, we have

$t_i^* = [(z-x_i)/D]\ T\ \forall i = 1,\ldots,q$

This, precisely, is what has earlier been alluded to as a 'PR-type' policy—the proportionality rule suggested by BF. It may be noted that if P_α^0 is the value of the poverty index before the transfers, then its value after the budgetary intervention is very simply given by $P_\alpha' = [(D - T)/D]P_\alpha^0$.

α = 1

Again, there is no unique solution for minimizing P_1 in problem 3: the set of optimal solutions for problem 3 is a strict subset of the set of optimal solutions for problem 2, and consists of all feasible allocations which exhaust the budget *and* respect the requirement that a poor individual never receives a share of the budget smaller than his share in the aggregate poverty deficit. Thus, each of the type-b, type-p, and PR-type policies is an optimal P_1-minimizing rule for problem 3. However, a type-e policy which is optimal for problem 2 is no longer so for problem 3. Briefly, if **B** is the set of optimal solutions for problem 1, **B′** the set of optimal solutions for problem 2, and **B″** the set of optimal solutions for problem 3, then $\mathbf{B''} \subset \mathbf{B'} \subset \mathbf{B}$.

α > 1

It is straightforward, again, that the type-p solution continues to remain the optimal solution for problem 3 too.

7 A summary of problems and solutions, and an example

A tabular summary, which sets out each of problems 1, 2 and 3 and the solutions to them for the P_α class of poverty indices, is presented below. It provides a compact resume of all the relevant results discussed in sections 3, 5 and 6 (the index P_1 is an exception, since it entails multiple solutions). To obtain a quantitative picture of the various allocational patterns entailed as optimal solutions to the problems we have considered, and to enable a view of how little or much these allocational patterns are weighted in favour of the poorest of the poor, a simple numerical example is also provided. In this example, the poverty line is taken to be Rs 50, and the budgetary outlay for transfers to the poor is assumed to be Rs 40. The income vector is taken to be represented by the three-person distribution of incomes (10,30,40). The summary table is completely self-explanatory, and will therefore not be elaborated on.

A summary of problems and solutions for the $P\alpha$ class of poverty indices, and a numerical example of poverty-minimizing allocations when $\{x_i\}_{i\in Q} = \{10,30,40\}$, z = 50, and T = 40

Problem	Poverty index	Optimal policy	Allocation t_1^*	t_2^*	t_3^*	Amount of budget utilized
1	P_0	Type-r	0	20	10	30
	$P_{\alpha\in(0,1)}$	Type-r+	10	20	10	40
	$P_{\alpha>1}$	Type-p	30	10	0	40
2	P_0	Type-b	40	0	0	40
	$P_{\alpha\in(0,1)}$	Type-e	15	15	10	40
	$P_{\alpha>1}$	Type-p	30	10	0	40
3	P_0	Type-b	40	0	0	40
	$P_{\alpha\in(0,1)}$	Type-PR	22.857	11.429	5.714	40
	$P_{\alpha>1}$	Type-p	30	10	0	40

8 Concluding observations

A poverty index is generally written as a normalized weighted sum of the income-gap ratios of the poor; the weights are either some function of the income-gap ratios themselves or some function of the rank-orders of the incomes. In either case, there appears to be no poverty index whose minimization under the constraints embodied in what may be called the canonical problem yields solutions different from what Bourguignon and Fields (1990) have called 'type-r', 'type-p' and 'mixed-type' solutions. In as much as the values of the policy-maker are expressed both through the minimand *and* the constraint structure of the optimization exercise s/he undertakes, this paper has sought to derive 'unconventional' allocational patterns as optimal solutions to extensions of the canonical problem in which 'egalitarian' values are incorporated into the constraints in a non-trivial manner. These allocational patterns are different from the three types of optimal budgetary strategy conventionally encountered in the literature. Some 'new' budgetary policies which emerge as solutions to variants of the canonical problem are ones which have been called type-b, type-e, and PR-type rules. The last of these—proposed by Bourguignon and Fields—is a particularly useful allocational rule, striking, as it does, a balance between the extremes of egalitarian and inegalitarian outcomes as dictated, respectively, by the type-p and type-r rules. The implications of the new budgetary rules for the claims of the poorer among the poor have also been briefly explicated. The question addressed in this paper, additionally, can be seen to resonate with the analytical structure of kindred questions which arise in the context of international aid allocational problems and Talmudic 'estate' problems. In this sense, therefore, there is more to the logical exercise reviewed here than the curiosity-value of a solution in search of a problem.

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