Concepts of Thermodynamics in Economic Systems

I. Economic Growth

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Abstract

Thermodynamics is a statistical theory for large atomic systems under constraints of energy. An economy is a large system of economic agents and goods under the constraints of capital. Both systems may be handled by the Lagrange principle, the law of statistics for large systems under constraints. Thermodynamics and economics are expected to follow the same concept:

1. First law of economics: profit is a non total differential form that depends on the path of acquisition.
2. Second law of economics: The mean capital or standard of living is the integrating factor of profit and leads to the entropy of capital distribution.
3. Third law of economics: work increases capital and reduces capital distribution. (work is related to collecting capital by distributing goods). Periodic work is always connected to two different economic levels.

Periodic production of industry and households leads to the Carnot process of monetary cycles, which determine economic growth. Supply and demand lead to Boltzmann distributions of capital (wealth in Germany 1993), of income (Germany, USA and Japan), and of goods (automobiles in Germany 1998).

Social bonds are equivalent to atomic bonds, they are attractive, repulsive or indifferent. Hierarchy, democracy and the global state correspond to solids, liquids and the gas state. Social interactions correspond to chemical reactions: intermarriage of Blacks and Whites in USA, Catholics and Protestants in Germany show the same phase diagrams as the gold – platinum system. In binary systems the Lagrange principle leads to the laws of six different interactions in socio-economic systems: partnership, hierarchy, equality, integration, segregation and aggression.

Classification codes: C1, E2, I5

Key words: Lagrange principle, production, economic growth, homogeneous systems, heterogeneous agents.

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1.0. Introduction

In the last ten years new interdisciplinary approaches to economics have developed in social and natural science. First steps have been made by W. Weidlich 1972, D. K. Foley 1994 , H. G. Stanley 1999, D. Helbing 1995, J. Mimkes 1995, Y. Aruka 2001, A. Matsumoto 2003 and others. In order to enhance the communication between different disciplines a number of international conferences have been carried out worldwide in the last five years, with topics on complexity in economics, econo-physics and economic evolution. The basic idea in this paper is the application of thermodynamic concepts to socio-economic systems. The idea actually goes back to Empedokles of Acragas (495–435 B. C.), who related the mixing of social groups to the solubility of liquids: *people, who love each other mix like water and wine; people, who hate each other segregate like water and oil.*

Thermodynamics is a statistical theory for large systems, which is actually based on two corresponding concepts: one approach is based on the first and second laws of thermodynamics and leads to "macro-physics" of motors, refrigerators and heat pumps, the other is the free energy concept, that leads to "micro-physics" of atomic interactions in physics, chemistry, metallurgy or meteorology.

1. According to the second law of thermodynamics

$$\delta W = d Y - T\, d \ln P \qquad (1.1).$$

Work (W) reduces entropy (ln P). In economics work reduces the entropy of capital distribution: business collects capital from customers by selling goods. This may be repeated by economic Carnot cycles and leads to economic growth. Eq.(1.1) leads "macro-economics", to production of industrial goods and monetary cycles. Work in thermodynamics and production in economics are the same. Industrial workers often become too expensive and are replaced by robots, computers, machines. These machines follow the second law of thermodynamics and often work with a higher efficiency than people.

2. The free energy concept is based on Lagrange principle of statistics,

$$L = T \ln P - Y \rightarrow \text{maximum !} \qquad (1.2)$$

Probability (P) is maximized under the constraint (Y), T is the Lagrange parameter. In thermodynamics L is the free energy, Y is the energy of atomic bonds, T is the mean kinetic energy or temperature. This concept may be translated to socio-economic systems with constraints and leads to "micro-economics". Economy is a market with traders under the constraint of prices. Society is a system of social agents under the constraints of social bonds. Statistical laws like Eq.(1.2) are never concerned with the type of object, but only with the number of objects. For this reason it will be necessary to discuss new meanings for the functions L, T, P and Y in social and economic systems. In atomic systems all thermal properties of materials (solids, liquids or gases) may be derived from the Lagrange principle, and it is the object of this paper to investigate, whether this concepts is also valid in other systems like economics, social science and politics.

1.1. Monetary cycles

In many text books on introductory economics (see Felderer Homburg 1999) the most simple relationship between industry and households is given by a monetary circle as shown in fig. 1.1. Industry pays wages Y (= 100) to the households, and the households consume the amount C (= 90) for goods. The money for consumption flows back to industry. The resulting savings S (= 10), the difference between wages Y and consumption C flows through the banks back to industry.

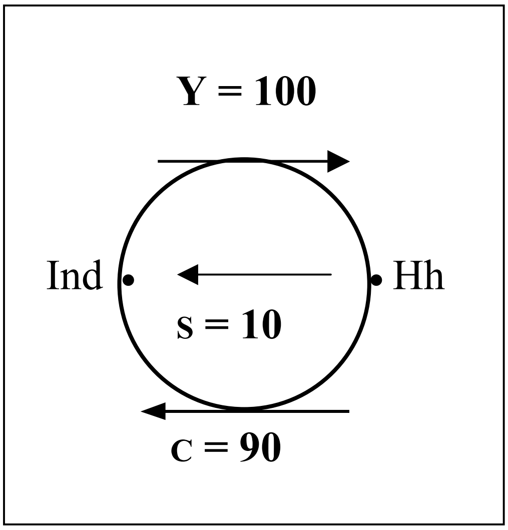

Fig.1.1. Traditional monetary cycle between industry (Ind) and households (Hh). The money flows by wages (Y) from industry to the households and from households back to industry by consumed goods (C) and savings (S).

The monetary cycle in fig. 1.1 leads to a reasonable result for normal households, they earn more than they consume and they may also save some money to make their capital grow.

However, fig.1.1 is not a reasonable cycle for the industry. According to the fig.1.1 the industry pays the households Y (= 100) in wages, gets back only C (= 90) by consumption and has to borrow S (= 10) from the bank in order to have at the end 100 % again. This is not the way industry works! And there is no industrial profit in this cycle! In reality industry pays C (= 100) in wages, expects to sell perhaps Y (= 120) to make a profit $\Delta Q = (Y - C) = 20$! The profit may then be reinvested. This is indicated by a separate cycle for the industry, fig.1.2.

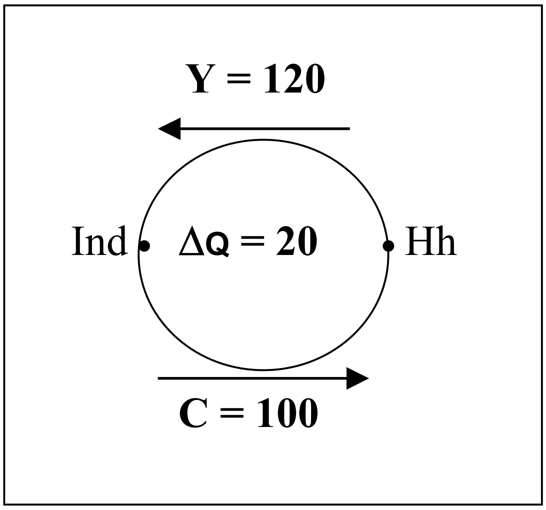

Fig. 1.2. Monetary cycle of industry. The money flows by wages from industry to households (C = 100) and from households back to industry by consumed goods (Y= 120, arbitrarily). The profit ($\Delta Q = Y - C = 20$) may be reinvested.

Figs. 1.1 and 1.2 indicate, that it may be necessary to take a separate look into the balance of each side, households and industry, before discussing a common cycle. The cycles in figs. 1.1 and 1.2 represent the economic balance for each unit, for each person, household, business, country or living creature. In each of these individual systems the income (Y) has to be higher than consumption (C). The profit ($\Delta Q = Y - C$) may be used in different ways, households may consume or save the profit, industries may reinvest, biological systems will use the profit for reproduction. The monetary cycle is closely related to the biological model of the blood stream, which distributes the energy to the body, and collects the waste on its way back, as proposed first by F. Quesnay (1694 – 1774). However, in contrast to the blood circuit the energy cycle in biological systems is not closed.

The mathematical description of this monetary cycle is closely related to the Carnot cycle of thermodynamics. This cycle makes it possible for engineers to run motors, refrigerators or heat pumps. The mathematical basis is given by the calculus of total and non total forms. These forms explain, that closed integrals or cycles are not always zero, but may lead to profit or loss of energy or capital.

1.2. Calculus of non total differential forms

To draw a cycle or a closed integral requires at least two dimensions, x and y. The closed line integral has the general form:

$$\oint a(x,y)dx + b(x,y)dy$$

A. Two-dimensional differential forms

$$d f = a(x, y) \, d x + b(x, y) \, d y \qquad (1.3)$$

are total differential forms, the function f exists and is given by the limits of integral, if

$$\partial a(x, y) / \partial y = \partial^2 f / \partial y \, \partial x = \partial^2 f / \partial x \, \partial y = \partial b(x, y) / \partial x. \qquad (1.4)$$

Since d f (x, y) depends on the limits of the integral, only, the closed integral is zero:

Example 1: $d f = (3 x^2 y^3) \, d x + (3 x^3 y^2) \, d y$

$$9 x^2 y^2 = \partial a(x, y) / \partial y = \partial^2 f / \partial y \, \partial x$$

$$= \partial^2 f / \partial x \, \partial y = \partial b(x, y) / \partial x = 9 x^2 y^2$$

$$f(x, y) = x^3 y^3 + \text{constant}$$

$$\oint df = 0 \qquad (1.5)$$

B. Two-dimensional differential forms

$$\delta \omega = a(x, y) \, d x + b(x, y) \, d y \qquad (1.6)$$

are non total differential forms, the function ω depends on the path of integration, if

$$\partial a(x, y) / \partial y \neq \partial b(x, y) / \partial x \qquad (1.7).$$

Since the integral depends on the path of integration, the closed integral is generally not zero,

$$\oint \delta\omega \neq 0 \qquad (1.8)$$

Example 2: $\delta \omega = (3 x^2 y^4) \, d x + (3 x^3 y^3) \, d y$

$$12 x^2 y^3 = \partial a(x, y) / \partial y \neq \partial b(x, y) / \partial x = 9 x^2 y^3$$

ω : *depends on the path of integration.*

1.3. First law of economics: profit

a) The first law of thermodynamics states that *heat δQ is a non total differential, the closed integral is not zero and the value of Q depends on the path of integration.,*

$$\oint \delta Q \neq 0 \qquad (1.9)$$

b) The same is true in economics. Investing money in Japan, in Europe or the US will lead to different profits in each case. The profit may also be negative, as losses are included. *Profit δQ is a non total differential, the value of profit depends on the way of investment.* Accordingly, Eq.(1.9) may be called the first law of economics.

1.4. Common monetary cycle of households and industry

Each individual and each company has its own monetary cycle and requires a positive integral of δQ in order to survive. But we can link the two cycles of industry and households on top of each other, as the income Y_{Hh} of the households is equal to the costs C_{Ind} of the industry. We now obtain a common cycle of industry and households, fig. 1.3. The common profit $\Delta Q_{Ind + Hh}$ has to be divided between industry and households by the representing agents (unions and industrial representatives).

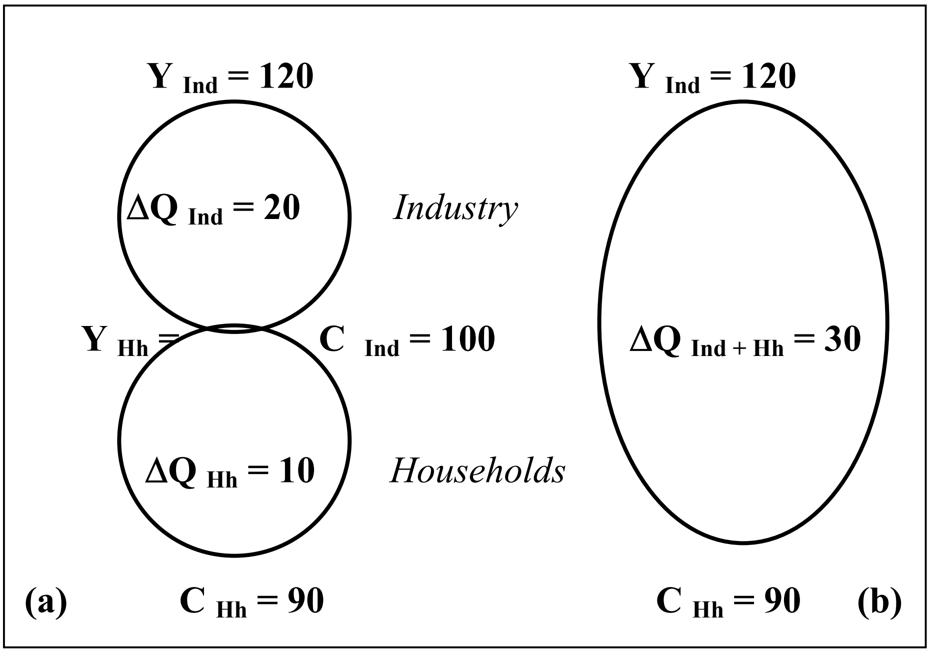

Fig. 1.3. Combined monetary cycle of industry (Ind) and households (Hh). By sales industry earns Y_{Ind} (= 120). The costs or wages C_{Ind} (= 100) are the income of the households, $Y_{Hh} = C_{Ind}$. The households, again only consume C_{Hh} (= 90). In the combined cycle the common profit ΔQ (= 30) has to be split between the two parties by their agents (unions and industrial representatives).

1.5. Economic chains

Like in fig. 1.3 monetary cycles of a product can be linked on top of each other, if we calculate the price (y = Y/N) per item. The cycles now form an economic chain. This is shown in fig. 1.4.

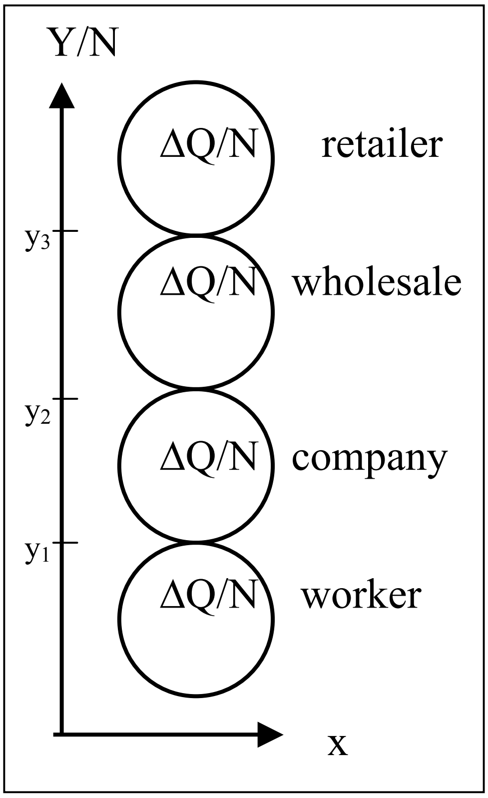

Fig. 1.4. Economic chain of monetary cycles for the production of N goods. The price per item (Y/N) will be different for workers, industry, whole sale and retailers. All groups are lined up in a hierarchy, each making profit $\Delta Q/N$ per item and adding more value to the product.

The worker produces a good and gets paid by the company and makes a profit at each product, y = Y/N. The company sells the product to the wholesaler with a profit. The retailer buys the product from the wholesaler and sells it with a profit to the customer. Each group makes a profit $\Delta Q/N$ per item and adds more value to each product. At the end the worker may not be able to afford the price of his product at the retailer. After each cycle the profit is divided by the economic agents (e.g. industry and unions, buyer and seller), the price y_n is determined by supply and demand.

A similar chain of monetary cycles may also be worked out for international trade. Over millions of years nature has acquired a large amount of resources like farming soil, oil, coal, gas or minerals. These resources lead to the mean wealth (Y/N) per household and are the basis of production by farming, mining especially in countries like Iran, Saudi Arabia, or Brazil. Industrial nations like Japan or Europe buy the resources from Iran or Brazil to produce new products of higher value, like machines, cars, computers, etc. The products are then sold at a high price, e.g. to the US, a market with a very high standard of living.

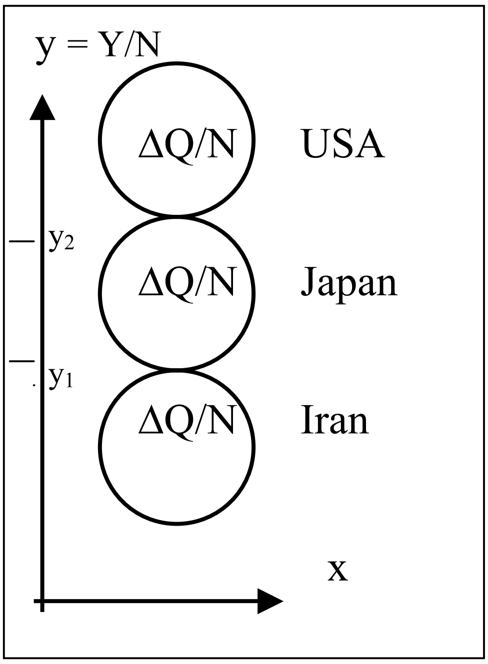

Fig. 1. 5. Economic chain of monetary cycles for different countries. All countries are lined up in a hierarchy of GDP per household or standard of living, each making profit and adding more value $\Delta Q/N$ to the product.

Fig. 1.5 reveals a problem: who will buy the products of the richest countries? One possible solutions is to sell the products to rich people in poor countries. But this mechanism will work only, if the number of poor countries exceeds the number of rich countries by far. Another solution is to own production means in poor countries. This will labor is exported to poor countries.

In figs. 1.4 and 1.5 the monetary cycles have been drawn in the x-y plane. The y axis in figs. 1.4 and 1.5 corresponds to price per item, $y = Y/N$ or capital per person. But so far the x – axis has not been defined. For this we have to go back to the calculus of non total differential forms.

1.6. Integration factor

C. A two-dimensional non total differential form (1.6)

$$\delta\omega = a(x, y)\,dx + b(x, y)\,dy$$

can be transformed into a total differential form d f by an integrating factor 1/y. The closed integral will be zero,

$$\oint \delta\omega / y = \oint df = 0 \qquad (1.10)$$

Example 3: $\delta\omega = (3x^2 y^4)\,dx + (3x^3 y^3)\,dy$ *(Example 2)*

$\delta\omega / y = (3x^2 y^3)\,dx + (3x^3 y^2)\,dy = df$ *(Example 1)*

The integrating factor of the non total differential form in Example 2 is given by 1/ y. This factor leads to the total differential form d f of example 1.

1.7. Second law of economics: The economic temperature

Second law of thermodynamics*: the temperature T is introduced as the integrating factor of the non total differential form of heat δ Q.* The temperature is the mean kinetic energy per atom, T = E / N.

Second law of economics: *economic temperature T is the integrating factor of the non total differential form of profit δ Q.* The economic temperature is the mean profit per person, T = Q / N. The economic temperature of a country is given by the mean Gross Domestic Product GDP per household (or capita) or standard of living, fig 1.6.a.

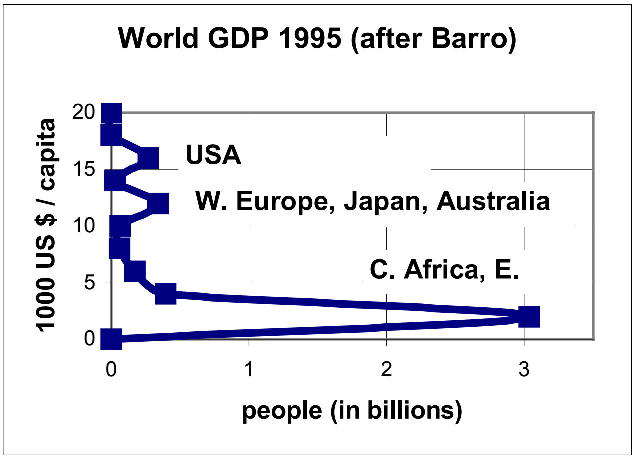

Fig. 1.6.a. The distribution of the world Gross National Product (GDP) in US $ per person (1995) after Barro and iMartin, 1995. The GDP / person is a measure of economic temperature (T) of different countries. N. America, W. Europe. Japan are rich nations and at a high standard of living, Central Africa and S. E Asia are very poor and at a low standard of living.

1.8. Entropy

The integrating factor 1 / y leads to a total differential form d f of a new function f. In thermodynamics the integrating factor 1 / T leads to the new function S,

$$dS = \delta Q / T \tag{1.11}$$

which is called entropy. The closed integral of entropy is zero. The closed integral of heat may now be written in terms of entropy:

$$\oint \delta Q = \oint TdS \neq 0 \tag{1.12}$$

The closed line integral Eq.(1.12) leads to profit, figs 4 and 5,

$$\Delta Q = \oint TdS = \oint ydx \neq 0 \tag{1.13}$$

The variables for monetary cycles (y, x) are given by the standard of living or wealth per household and by entropy, y = T = Y / N and x = S. Entropy (S) has already been introduced to economics by Roegen. However, the introduction did not show the vital importance of entropy in economics.

In thermodynamics entropy is closely connected to the probability (P) of energy distribution in a system like a gas,

$$S = \ln P \qquad (1.14),$$

$$P = N! / (N_1 ! N_2 ! ...N_k !) / K^N \qquad (1.15).$$

In economic systems the entropy is closely connected to the capital distribution in an economic system like a market. This may be shown by a simple example:

Example 4: *A farmer sells ten pounds of apples at a price of 1 € per pound*

A. *Before selling the apples to his ten customers the ten 1 €- coins are evenly distributed among the ten customers and none at the farmer, fig. 7 a.*

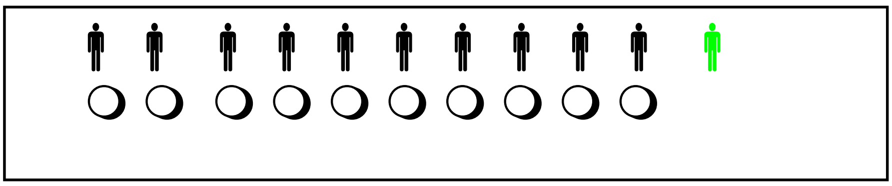

Fig. 1.7 a. *Before the sale the ten 1 €- coins are evenly distributed among the ten customers and none with the farmer.*

The probability of a distribution of ten 1 €- coins at ten customers and none with the farmer is given by Eq. (1.16) for N = 10 and K = 11:

$$P_1 = 10! / (1! \, 1! \, ... \, 1! \, 0!) / 11^{10} = 0{,}00014 \tag{1.16}$$

$$S_1 = \ln (0{,}00036288) = - 8{,}87 \tag{1.17}$$

The entropy of this distribution is negative, since the probability is always P < 1.

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***B.** After selling ten pounds of apples for 1 € each, the ten 1€-coins are now distributed in a new way: one person (the farmer) has all coins, all other ten customers have no more coins, fig. 7 b.*

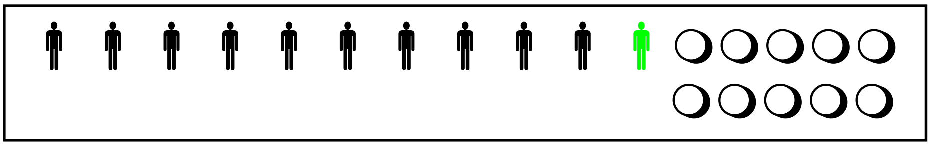

***Fig. 1.7 b.** After the sale the ten 1 €- coins are distributed unevenly, the farmer has all ten 1 €- coins, the ten customers have no coins.*

The probability of a distribution of ten 1 € coins at one out of 11 person is given by

$$P_2 = 10! / (10!\,0!\,0!\,...\,0!) / 11^{10} = 11^{-10} \qquad (1.18)$$

$$\mathbf{S_2 = -10\ \ln 11 = -23{,}98} \qquad (1.19)$$

In the process of selling the entropy of money distribution has changed by

$$S_2 - S_1 = -23{,}98 + 8{,}87 = -15{,}11 \qquad (1.20)$$

This result shows the importance of entropy in economics:

Collecting (goods, money) is equivalent to reducing entropy.
Distribution (of goods, money) is equivalent to increasing entropy!

If we look at buying and selling, entropy goes just the opposite way for money and goods. Buying means collecting goods and distributing money. Selling means distributing goods and collecting of money. The flows of products and money are opposite. Changing entropy is the important mechanism in buying and selling.

***Example 5:** Why can't we do business by cutting each others hair? Or pay a kg of apples by a kg of apples? It is not, that hair cutting is no work. And a kg of apples is a value that may very well serve in trade. The answer is: Trading two identical items does not change the entropy!! The distribution of apples is the same before and after the trade. Trade means exchange of different items!*
***Example 6:** Why is money so important in trade? Money is always different from the objects we are buying or selling. Paying with money always changes the entropy. Money also can be portioned, so it will always have the same trading value as the object we want to buy or sell. For this reason we cannot use life stock as currency. But gold or Kauri shells will also work as currency. But we cannot do business by buying or selling money for the same amount of money or trade Kauri shells for Kauri shells.*

2.0. Third law of economics: work

Without work we have dissipation of entropy:

Example 7: *Water flows from high to low (in a river)*
Heat flows from warm to cold (in a furnace)
Capital flows from rich to poor (in welfare)

Work can reverse the process of entropy dissipation:

Example 8: *With work water also flows from low to high (in a water pump)*
With work heat also flows from cold to warm (in a heat pump)
With work capital also flows from poor to rich (in business)

The first and second laws of thermodynamics (1.1) may be written as

$$\delta W = dY - T\,dS \qquad (2.1)$$

Work increases energy (Y) and reduces energy distribution (S). Replacing energy by capital leads to the third law of economics, Eq.(2.1): *work increases capital (Y) and reduces capital dissipation (S).* This has been indicated in fig. 7 b, selling apples is work and leads to the collection of capital. In economics work is equivalent to increasing and collecting capital.

2.1. Carnot cycles

Cyclic work of a motor leads to the Carnot cycle and may be calculated by the closed line integral

$$-\oint \delta W = -\oint (dY - TdS) = \oint \delta Q = Q = \Delta T \Delta S \qquad (2.2)$$

As the closed line integral of the total differential form d Y is zero, only the non total part δQ = T d S remains in Eq.(2.1).

In a motor heat (Q) is changed into work (W), in a heat pump work (W) is turned into heat (Q). Machines like motors or heat pumps always require or create two different temperatures (T): Inside the motor it is hot and needs water or air cooling, outside. The heat pump works with a cold river and a warm house, the refrigerator has a cold inside and a warm outside. This difference in temperature is necessary to make the machine work. If the door of the refrigerator stays open, it will not work.

Accordingly, labor, business and trade always require or create two different price levels or standards of living (T), costs (C) have to be lower than income (Y). If costs and income are the same, a company or household cannot work. Fig. 2.1 demonstrates the Carnot cycle and corresponds the monetary cycle in fig 1.2. At (Y) a high amount of money is collected for the goods from the customers. At (C) a low amount of money is distributed to the workers for production of goods. In more detail the Carnot process has four components: production (C), import, sales (Y) and export.

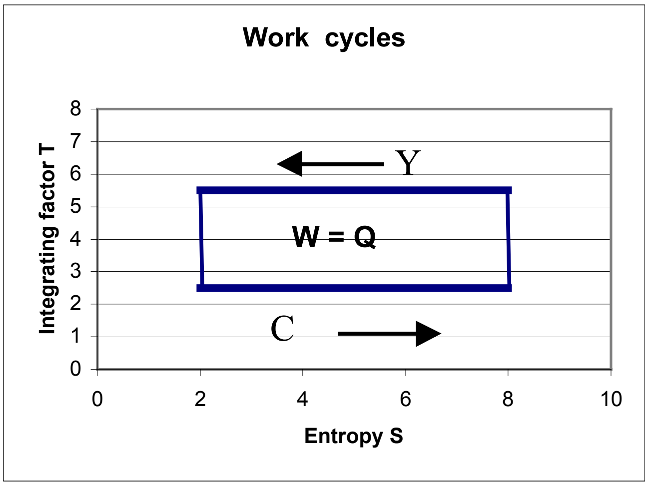

Fig. 2.1. The Carnot process of a motor in a T – S diagram. The energy is collected at high temperature from the heat of combustion (at Y) and is distributed by the exhaust to the cold air (at C). The process corresponds to the monetary cycle in fig. 1.2: At Y a high amount of money is collected for the goods from the customers. At C a low amount of money is distributed to the workers for production of goods.

2.1.1. Production in a poorer country

In the production process of cars or computers (C in fig. 2.1) the components do not change their value, $Y = Y_0$,

$$\delta W = d Y_0 - T d S = - T d S \qquad (2.1a)$$

$$W = - T \Delta S = - T \Delta \ln P \qquad (2.1b)$$

Production is equivalent to reducing entropy ($\Delta S = \Delta \ln P$) of the components. The value (Y) of the material remains constant in the production process.

Example 9: *A car or computer is produced by fitting all components together. Before production the components could be arranged in many ways, the number of possibilities and the entropy are very high. In the process of production the components are put together according to the production rules in the one and only possible way. The number of possibilities now is $P = 1$, and the entropy is very small, $\ln P = 0$. Production is equivalent to ordering and reducing the entropy ΔS of the components. The value (Y) of the components remains unchanged.*

In Eq.(2.1a) wages are proportional to the standard of living (T): the higher the standard of living, the higher the costs of labor (W). In the production of a certain brand of cars or computers the workers have to follow the same production rules (ΔS) in countries with high or low standards of living. But for the same product wages are low in countries with low standard of living, $W_1 = T_1 \Delta S$, and high in countries with high standard of living, $W_2 = T_2 \Delta S$.

In the process of production goods are collected and money is distributed to workers at a low level of income y. High production leads to a low level of unemployed people in the poorer country.

2.1.2. Import from a poorer to a richer country

Import does not change the entropy of the components of cars or computers, import is work at constant entropy (S_0),

$$\delta W = dY - T\,dS_0 = dY \qquad (2.1c)$$

$$W = \Delta Y \qquad (2.1d)$$

Example 10: *After fabrication Indian cloth or Chinese furniture is exported to Europe, where it sells for a higher price.*

Example 11: *After catching fish a fisherman cannot sell the fish to his fellow fishermen. He has to bring the fish to a market, where the price of fish is higher than his own production costs.*

Import is equivalent to collecting goods and distributing money between two levels of income Y. Import of goods makes the poorer country richer and the richer country poorer. Import of goods is also an export of jobs and increases unemployment in the richer country.

2.1.3. Sales in a richer country

Sales are equivalent to distributing goods and collecting capital from the customers at high level of income (Y in fig. 2.1). Sales are positive for employment. However, a high level of income is a problem for production and income. Where does the money come from and who will buy the products of a rich country? Rich countries will mainly have trade with other rich countries. But economic growth is not possible for trade at constant economic level Y, there is only exchange of different products at the same price. A real profit always requires two different levels of price or income.

2.1.4. Export from a richer to a poorer country

Export from a rich country to a poor country does not change the value of the product!

Example 13: *After fabrication of an expensive car in a high level country it may be exported to a country with a low standard of living. As the value of the car is not changed during export most people in a low level country will not be able to buy this car. Only rich persons in a poor country will be able to buy the car.*

By exporting goods from a richer to a poorer country capital is collected (extracted) from the poor county. The poor country becomes poorer, the rich country becomes richer. End exports are positive for employment.

2.2. Efficiency, returns

Profits may also be increased by a larger difference ΔT or efficiency ΔT / T. The ideal efficiency or returns (r) of work per cycle is determined by the rate of profit and investment, (Q : C),

$$r = Q / C = \Delta T \; \Delta S / \; T_1 \Delta S = \Delta T / T_1 \qquad (2.3)$$

Eq.(2.3) may be explained by a calculation of profit in international trade. Fig. 1.6b shows the standard of living (T) or personal income (GDP per capita) for the world population in 1995 US $ after Barro and iMartin 1995. In N. America, W. Europe, Japan and Australia the a mean standard of living is close to T = 12.000 US $ per year, it is marked by a dashed line. The standard of living (T) of the poor countries in C. Africa or C. Asia is close to T = 2.500 US $ per person and is marked by a pointed line. Trade between these two different income zones is discussed in example 14.

Example 14: *The import of goods like furniture from Indonesia to Europe leads to a profit that can be calculated according to the lines in fig. 6b and Eq. (2.3):*

W. Europe : T_2 *= 12.000 US $ / person*

Indonesia T_1 *= 3.000 US $ / person*

$r = \Delta T / T_1 = (12.000 - 3.000) / 3.000) = 3 = 300\,\%$

The ideal efficiency of imports from Indonesia to W. Europe is r = 300 %. For each dollar or euro invested in Indonesian labor the (idealized) returns will be about 300 %. The returns are ideally independent of the item produced.

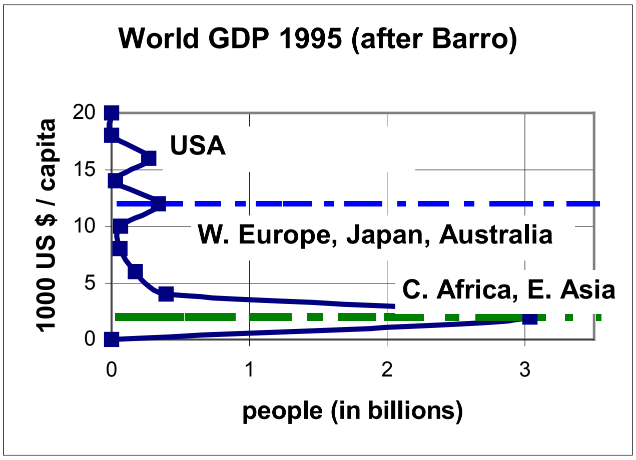

Fig.1. 6b. The distribution of the world Gross National Product (GDP) in US $ per person (1995) after Barro and iMartin, 1995. The GDP / person is a measure of economic temperature (T) of different countries. N. America, W. Europe. Japan are rich nations and at a high standard of living (upper line), Central Africa and S. E Asia are very poor and at a low standard of living (lower line).

2.3. Exponential growth

The definition of returns, Eq.(2.3), may be written as

$$Q = r\ C \quad \text{or} \quad \Delta Y = r\ Y \qquad (2.3a).$$

Profit (Q) is due to invested capital (Y) and depends on the efficiency or returns (r) of work. Profits may be raised by increasing the capital or increasing the efficiency of work. In saving accounts r corresponds to the interest per cycle.

The equations of thermodynamics (1.1, 1.2) are based on probability, but generally they do not include time. This makes a statistical theory like thermodynamics less attractive for economists, who want to know what happens tomorrow. However, the application of cyclic work makes it possible to introduce time (t) in units of the length of the cycle. Eq.(2.3 a) may be written in terms of time:

$$d\,Y = r(t)\ Y(t)\ d\,t \qquad (2.4)$$

or

$$d \ln Y(t) = r(t)\, d\,t.$$

At a constant rate of interest η at infinitely small cycles a bank account Y(t) shows exponential growth with time,

$$Y(t) = Y_0 \exp(r\,t) \qquad (2.5).$$

The same applies to the value of shares and the growth of economies. Exponential growth has been observed in many economies in the last century.

2. 4. Two interdependent economic systems

The Carnot cycle in fig. 2.1 corresponds to fig 1.3 and may be interpreted by two interdependent economic systems like industry and households, or first and third world. According to Eq.(2.4) the common profit is determined by the economic economic cycle in fig. 2.1 and the levels $C=Y_1$ and Y_2 of each party. At the end of the cycle the profit is shared between the two parties by the corresponding economic agents. If the profit is reinvested, the economic levels Y_1 and Y_2 of both parties will rise, accordingly. If the first party at lower level (Y_1) gets the share "p" and the second party at higher level (Y_2) gets a share (1 – p) of the profit Q, we obtain:

$$d\,Y_1(t) = p\ Q\ d\,t = p\ (\Delta S / N_1)(Y_2 - Y_1)\ d\,t \qquad (2.6)$$

$$d\,Y_2(t) = (1-p)\ Q\ d\,t = (1-p)\ (\Delta S / N_2)(Y_2 - Y_1)\ d\,t \qquad (2.7).$$

The constants N_1 and N_2 reflect the number of people of the systems (1) and (2). The solution of this set of differential equations (for $p \neq N_1/(N_1+N_2)$) is:

$$Y_1(t) = Y_{10} + p\,[Y_{20} - Y_{10}]\,[\exp(\alpha\,t) - 1]\ \Delta S / (N_1\ \alpha) \qquad (2.8).$$

$$Y_2(t) = Y_{20} + (1-p)\,[Y_{20} - Y_{10}]\,[\exp(\alpha\,t) - 1]\ \Delta S / (N_2\ \alpha) \qquad (2.9),$$

with $\alpha = (1-p)\,\Delta S / N_2 - p\,\Delta S / N_1$

The process of economic growth is indicated in fig. 2.2. The first cycle (dashed lines) has resulted in a profit ΔQ. After distributing the profit ΔQ the Carnot process starts again with a new upper level Y_2 and a new lower level Y_1 (solid lines), as indicated in fig. 2.2. In each new economic cycle we will have new levels for Y and C and a new profit $\Delta Q = Y - C$.

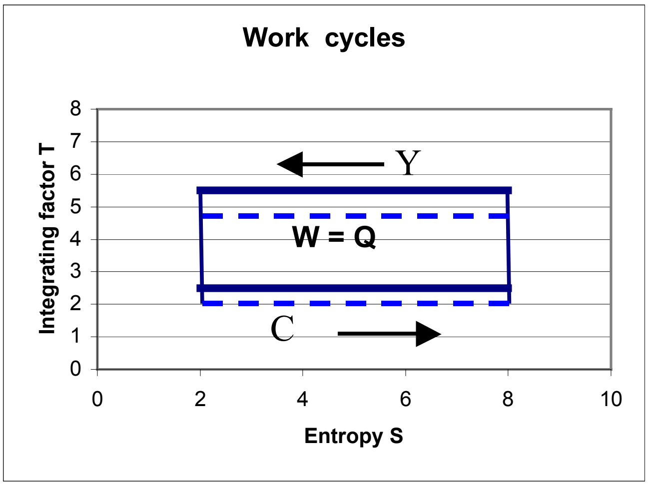

Fig. 2.2. The process of economic growth. The profit ΔQ of the first Carnot cycle (dashed lines) is distributed to the two levels by the economic agents and leads to new levels Y and C (solid lines). The next Carnot cycle starts from the new levels.

The solution for $p \neq N_1/(N_1+N_2)$ is given by:

$$Y_1(t) = Y_{10} + p\ t\,[Y_{20} - Y_{10}]\quad \Delta S / N_1 \qquad (2.10)$$

$$Y_2(t) = Y_{20} + (1\text{-}p)\ t\,[Y_{20} - Y_{10}]\quad \Delta S / N_2 \qquad (2.11).$$

According to Eq.(2.6) to (2.11) a rising standard of living (Y) in two interdependent economic systems is determined by the share of the profit “p” of the group at the lower level (Y_1). The results are shown in figs. 2.3 to 2.9. Defining the parameter $a = p\ N_1 / (N_1 + N_1)$ we find:

1. a) = 0; fig. 2.3: If all profit goes to the richer party (Y_2), the standard of living of group (2) will grow exponentially, the standard of living of the first party stays constant, (Y_{10}) .
2. a = 0,25; fig. 2.3: at 25 % of the profit for the poorer party (Y_1) and 75 % for the rich party (Y_2) both parties will grow exponentially. Examples are Japan and Germany after World War II, both economies were depending on the US and were growing exponentially, this is indicated in fig. 2.4.
3. a = 0,50; fig. 2.3: An even split between the two parties leads to a linear growth of both parties. The efficiency of the interaction is reduced with time.
4. a = 0,75; fig. 2.5: The growth of both parties is leveling off not much above the initial standard of living. An example is the present US-Japanese economic relationship, both economies are close to each other without much economic growth, as shown in fig. 2.6.
5. a = 1,00; If all profit goes to the poor side, the standard of living of the poor party soon reaches the constant standard of living of the rich party.

6. a = 1,25; fig. 2.7: If more than 100 % of the profit goes to the poor party, (Y_2) will decrease, and (Y_1) will catch up with Y_2). This has been observed in the relationship of West and East Germany after reunion in 1990, fig. 2.8.
7. a = - 0,25; fig. 2.9: If the poor side (Y_1) makes only losses and will go bankrupt. The richer party (Y_2) will grow exponentially. This has been observed in Argentina, where many rich people transferred there assets to the US, fig. 2.10.

The data in figs. 2.4, to 2.10 can only indicate the results of Eqs.(2.8 to 2.11), as all countries also have other (less important) interactions with other countries. The results may also be applied to other binary interactive economic systems like industry and households or in trade. For industries and households the distribution of profit p is determined by the interacting agents of unions and industry, in trade we have buyer and seller. This is now discussed in more detail.

2.4.1. Exponential growth, industry and unions (0 <p < 0,5)

Fig 2.4 shows the problem of unions and industry in more detail. Unions tend to ask for high raises in payments, industry urges to invest the profits. Indeed, the fair deal, a split of profits 50 : 50 between workers and industry (green line) in fig. 2.1 is not the best deal and will only result in linear growth. Workers and industry are much better off by a deal, where 90 % of the profits are reinvested (red lines). This is shown in fig. 2.4. Low increase of wages will lead to exponential growth for industry and workers. But workers (as well as their managers) will have to be more patient with pay raises, like in Germany or Japan after world war II, fig. 2.5.

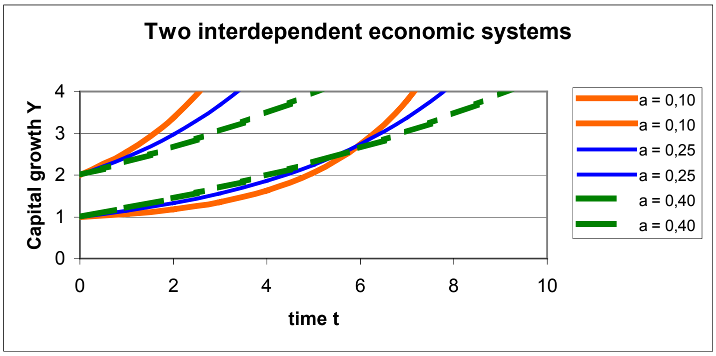

Fig. 2.3. *The development of standard of living of two interdependent economic systems stating at* $Y_1 = 1$ *and* $Y_2 = 2$. *The profit for the poor side varies from p = 0,10 to p = 0,40. After some time the standard of living of workers* (Y_1) *will grow with lower pay raise p !!*

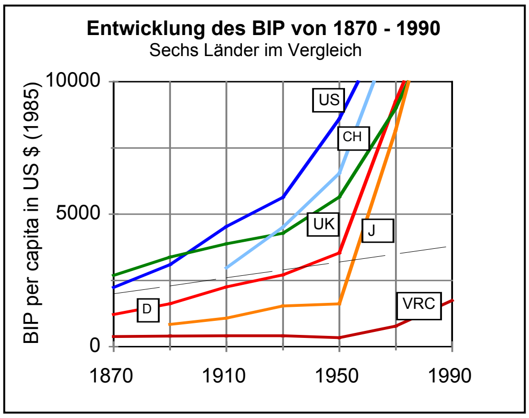

Fig. 2.4. *Economic growth of US, UK, Switzerland, Japan, Germany, China between 1870 and 1990 (after Barro and iMartin 1995). The victorious allies USA and UK have grown exponentially. Japan and Germany only started to grow exponentially after World War II by international trade at low wages. China was excluded and did not take part in economic growth, then.*

2.4.2. Trailing economies: USA – Japan (0,5 < p < 1)

The opposite picture is shown in fig. 2.5. A high factor p leads to decreasing efficiency, (Y_1) is trailing a decreasing (Y_2). After Japan and Germany have acquired many production plants, the factor p has grown and the efficiency of the exports started to decrease. In fig. 2.6 the economic level (Y_1) of Japan now is trailing the slowly decreasing level (Y_2) of the USA.

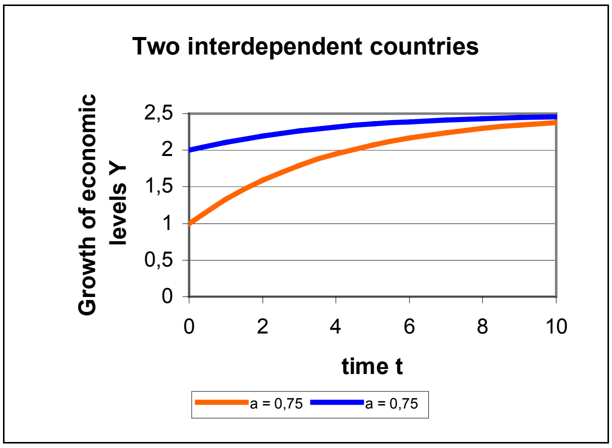

Fig. 2.5. *The development of standard of living of two interdependent economic systems stating at $Y_1 = 1$ and $Y_2 = 2$. At high values of profit for the poor side, p = 0,75 , economic growth is declining with time.*

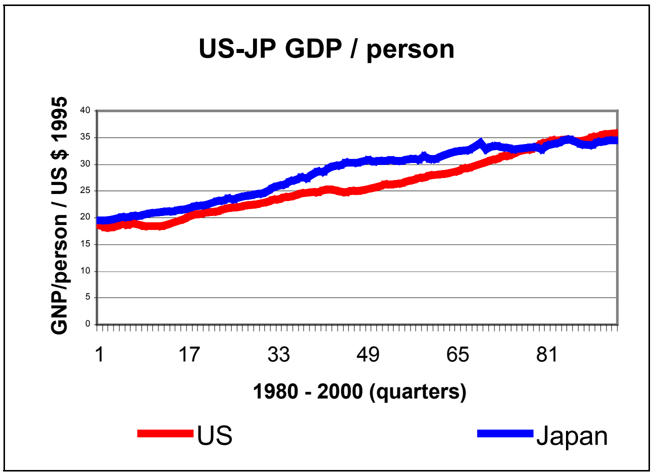

Fig. 2.6. *The development of standard of living (GDP / person) of the USA and Japan in quarters between 1980 and 2000.The interdependent economic systems are declining with time.*

2.4.3. Converging economies, West and East Germany (p > 1)

If the poor side (Y_1) profits very much, $a = p N_1 / (N_1 + N_1) > 1$, both parties will converge, as shown in fig. 2.7. This happened during the reunification of West and East Germany, fig. 2.8. The standard of living in East Germany grew by 100 % within six years, as the standard of living in West Germany was declining. The economic levels (Y_1) and (Y_2) in East and West Germany have nearly converged and differ now after more than 15 years by only 20 %.

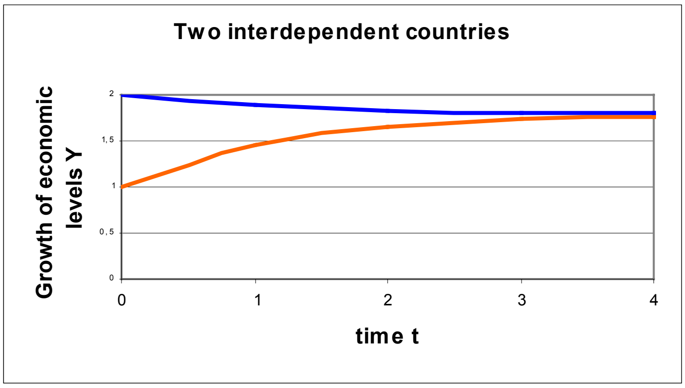

Fig. 2.7. *The development of standard of living of two interdependent economic systems stating at $Y_1 = 1$ and $Y_{2.} = 2$. At very high values of profit for the poor side, $p > 1$ both economies will converge below $Y_{2.} = 2$.*

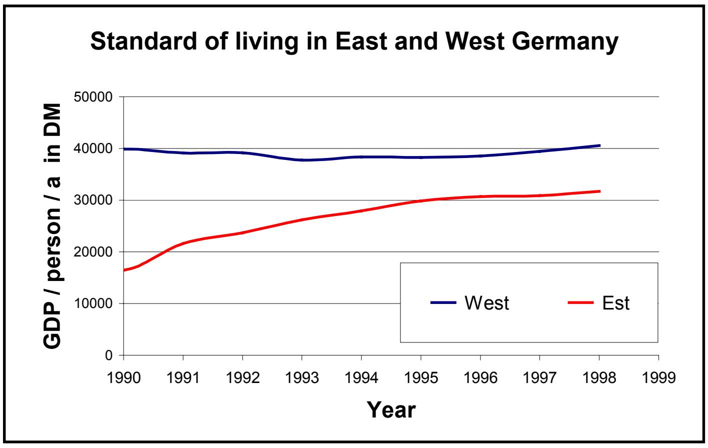

Fig. 2.8. *Real standard of living in West and Germany between 1989 and 1998 due to productivity and capital transfer. In 1998 East Germany reached about 80 % of the living standard in West Germany (Fründ 2002).*

2.4.4. Exploitation ($p < 0$)

In the last 500 years European countries have exploited their colonies. Spain took the gold from Central and South America, England exploited India, Holland conquered Indonesia etc. Fig 2.9 shows the development of two economies for negative profit, $a = p\ N_1 / (N_1 + N_1) = -0{,}25$. The dominating country shows exponential growth, the exploited country breaks down, the standard of living tends toward zero. A modern example is the relationship USA – Argentina from 1996 on, where large amounts of capital were transferred to the US, which effected the economy of Argentina, considerably, fig. 2.10.

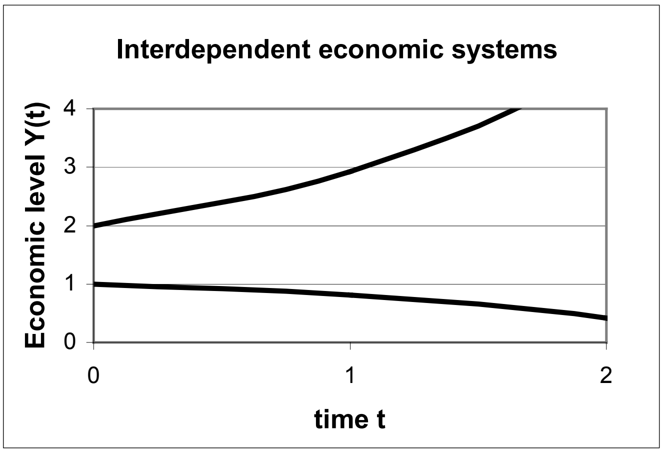

Fig.2.9. *The development of standard of living of two interdependent economic systems stating at $Y_1 = 1$ and $Y_2 = 2$. The poor side is exploited, the profit is negative, $a = p\ N_1 / (N_1 + N_1) = -0{,}25$. The economic level of the dominating party grows exponentially, the exploited party falters.*

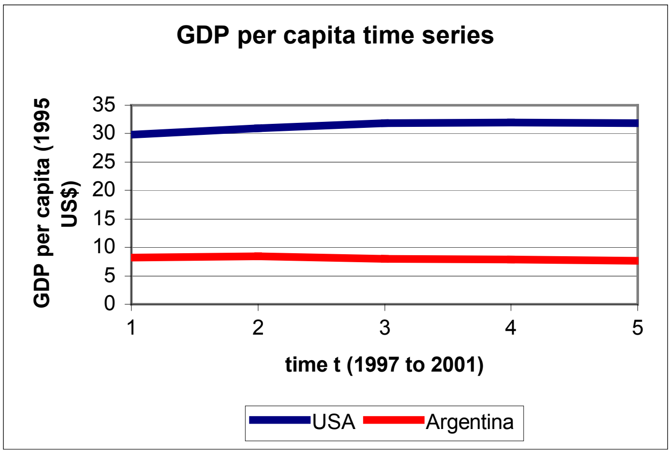

Fig. 2.10. *The development of standard of living of USA and Argentina between 1997 and 2001. Due to the large amounts of capital transferred to the US, the Argentina standard of living was reduced, considerably..*

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